When Cohomology Meets Linearity

AOPs in Therapy

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New symmetric primitives

What are AOPs?

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Appellations d'Origine Protégée ?

Munster

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Arithmetization-Oriented Primitives!

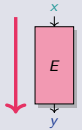
New symmetric primitives for

- ⋆ **MPC**: Multiparty Computation
- ⋆ **FHE**: Fully Homomorphic Encryption
- ⋆ **ZK**: Systems of Zero-Knowledge proofs

Comparison with the traditional case

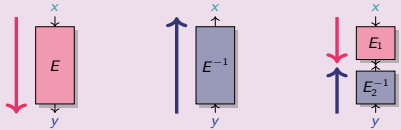
Traditional case

$y \leftarrow E(x)$



Arithmetization-oriented

$y \leftarrow E(x)$ and $y == E(x)$



Comparison with the traditional case

Traditional case

$$y \leftarrow E(x)$$

- ⋆ Optimized for:
  implementation in software/hardware

Arithmetization-oriented

$$y \leftarrow E(x) \quad \text{and} \quad y == E(x)$$

- ⋆ Optimized for:
  integration within advanced protocols

Comparison with the traditional case

Traditional case

$$y \leftarrow E(x)$$

- ⋆ Optimized for:
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- ⋆ Alphabet size:
  $\mathbb{F}_2^n$, with $n \simeq 4, 8$

  Ex: Field of AES: $\mathbb{F}_{2^n}$ where $n = 8$

Arithmetization-oriented

$$y \leftarrow E(x) \quad \text{and} \quad y == E(x)$$

- ⋆ Optimized for:
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- ⋆ Alphabet size:
  $\mathbb{F}_q$, with $q \in \{2^n, p\}, p \simeq 2^n$, $n \geq 64$

  Ex: Scalar Field of Curve BLS12-381: $\mathbb{F}_p$ where

  $p =$ 0x73eda753299d7d483339d80809a1d805
  53bda402fffe5bfeffffffff00000001

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Cryptanalysis

Decades of analysis

Arithmetization-oriented

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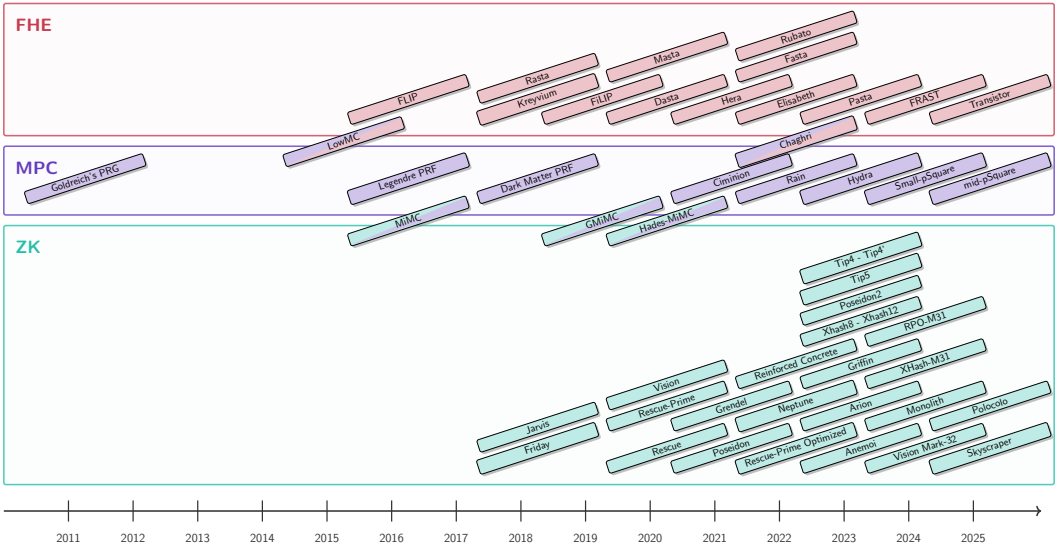
Cryptanalysis

≤ 8 years of analysis

Many new symmetric primitives



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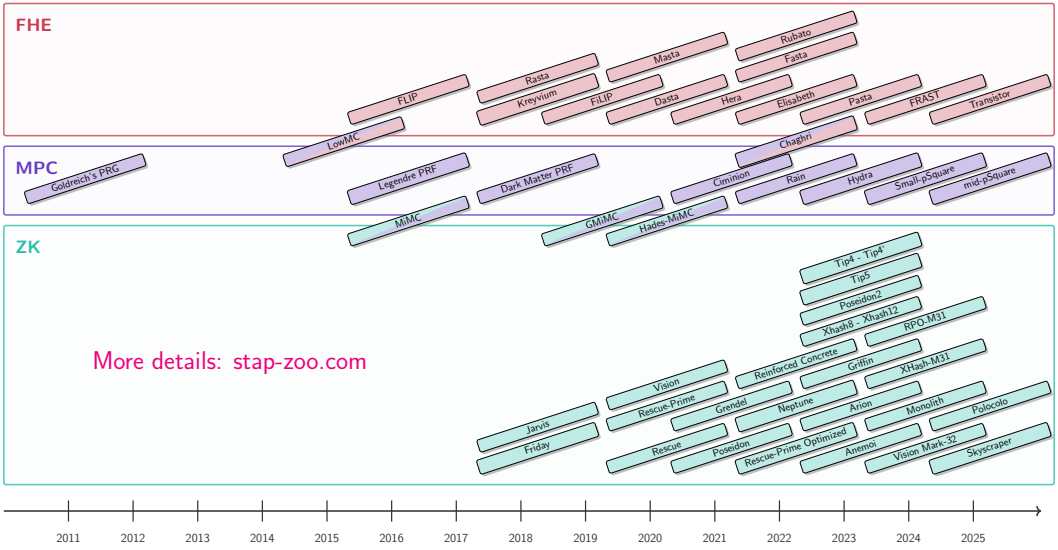
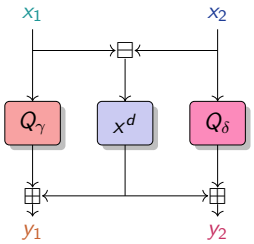


Past works



Closed Flystel: $F : \mathbb{F}_q^2 \to \mathbb{F}_q^2, (x_1, x_2) \mapsto (y_1, y_2)$

- ⋆ Introduced by [C. Bouvier, P. Briaud, P. Chaidos, L. Perrin, R. Salen, V. Velichkov and D. Willems, CRYPTO 2023]
- ⋆ Building block of Anemoi
- ⋆ Degenerated case of Butterfly

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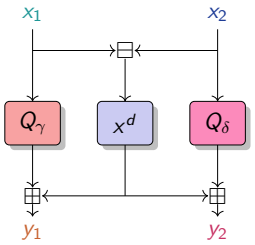

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Solving an open problem since 2014 on APN permutations over $\mathbb{F}_p^2$

Existence of APN permutations

F is Almost Perfect Nonlinear iff

$$\delta_F = \max_{a \neq 0, b} |\{x \in \mathbb{F}_p^m, F(x+a) - F(x) = b\}| = 2 .$$

Flystel in $\mathbb{F}_p$ with x^d: $\delta_{Flystel} \leq d - 1$

Recent cryptanalysis progress

- ⋆ Direct applications of results for exponential sums [T. Beyne and C. Bouvier, 2024]
- ⋆ Solving a conjecture on the linearity of the Flystel construction

Linearity bound

$$\mathcal{L}_{Flystel} = \max_{u \neq 0, v} \left| \sum_{x \in \mathbb{F}_p^2} e^{(2i\pi/p)(\langle v, F(x) \rangle - \langle u, x \rangle)} \right| \leq (d-1)p$$

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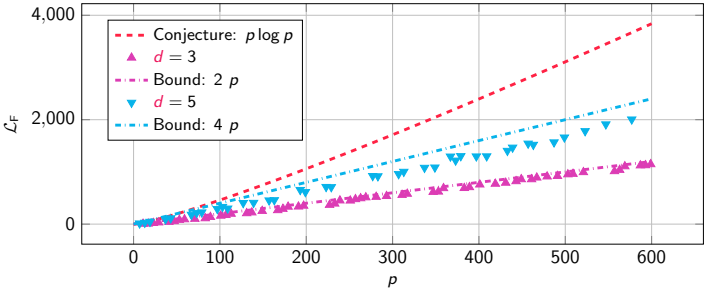

Work in progress

- ⋆ Can we provide detailed calculations of the cohomological spaces to refine bounds on the linearity of Butterfly-like constructions?
  (with C. Levrat)
- ⋆ Can we generalize to other constructions?
- ⋆ Is the Flystel an optimal construction? How to classify Butterfly-like constructions?
  [C. Bouvier, Fq 2025]

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Thank you!