# Anemoi: Exploiting the Link between Arithmetization-Orientation and CCZ-equivalence

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joint work with Pierre Briaud[1,2], Pyrros Chaidos[3], Léo Perrin[2], Robin Salen[4], Vesselin Velichkov[5,6] and Danny Willems[7,8]

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[5]University of Edinburgh, [6]Clearmatics, London, [7]Nomadic Labs, Paris, [8]Inria and LIX, CNRS

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# Why Anemoi?

- **Anemoi:** Greek gods of winds

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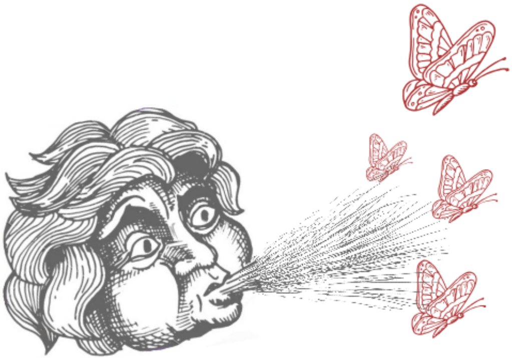

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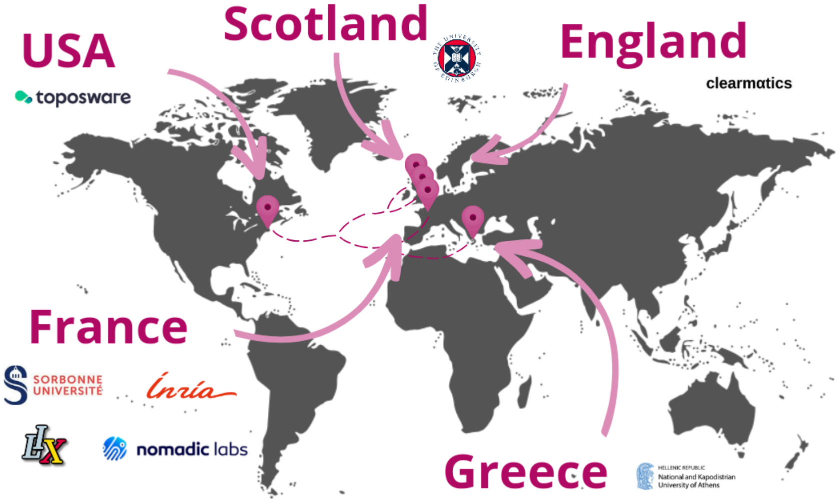

# Why Anemoi?

⋆ Anemoi: Family of ZK-friendly Hash functions

# Content

## Anemoi: Exploiting the Link between Arithmetization-Orientation and CCZ-equivalence

# A need of new symmetric primitives

Protocols requiring new primitives:

- ⋆ **MPC**: Multiparty Computation
- ⋆ **FHE**: Fully Homomorphic Encryption
- ⋆ **ZK**: Systems of Zero-Knowledge proofs

  Example: SNARKs, STARKs, Bulletproofs

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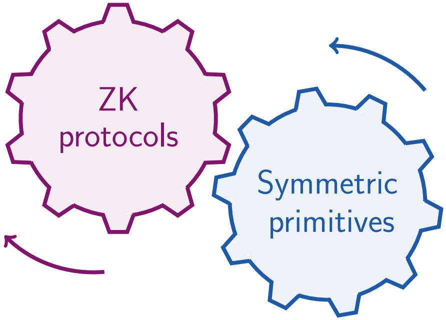


**Need**: Designing ZK-friendly symmetric primitives

$\Rightarrow$ What differs from the “usual” case?

# Comparison with "usual" case

## A new environment

**"Usual" case**

- Field size:
  $\mathbb{F}_{2^n}$, with $n \simeq 4, 8$
- Operations:
  logical gates/CPU instructions

**Arithmetization-friendly**

- Field size:
  $\mathbb{F}_q$, with $q \in \{2^n, p\}, p \simeq 2^n$, $n \geq 64$
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Ex: Field of AES: $\mathbb{F}_{2^n}$ where $n = 8$

Ex: Scalar Field of Curve BLS12-381: $\mathbb{F}_p$ where

$$p = \texttt{0x73eda753299d7d483339d80809a1d805}$$
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## New properties

**"Usual" case**

$$y \leftarrow E(x)$$

- Optimized for:
  implementation in software/hardware

**Arithmetization-friendly**

$$y \leftarrow E(x) \quad \text{and} \quad y == E(x)$$

- Optimized for:
  integration within advanced protocols

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Example: Minimize the number of multiplications (R1CS)

$$y = (ax + b)^3 (cx + d) + ex$$

$$t_0 = a \cdot x \qquad t_3 = t_2 \times t_1 \qquad t_6 = t_3 \times t_5$$
$$t_1 = t_0 + b \qquad t_4 = c \cdot x \qquad t_7 = e \cdot x$$
$$t_2 = t_1 \times t_1 \qquad t_5 = t_4 + d \qquad t_8 = t_6 + t_7$$

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3 constraints

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- **Our approach:** using $(u, v) = \mathcal{L}(x, y)$

$y \leftarrow F(x)$ $\rightsquigarrow$ $F$: high degree

$v == G(u)$ $\rightsquigarrow$ $G$: low degree

# CCZ-equivalence

Example: the inverse

$$\Gamma_F = \{(x, F(x)), x \in \mathbb{F}_q\} \quad \text{and} \quad \Gamma_{F^{-1}} = \left\{\left(y, F^{-1}(y)\right), y \in \mathbb{F}_q\right\}$$

Noting that

$$\Gamma_F = \left\{\left(F^{-1}(y), y\right), y \in \mathbb{F}_q\right\} \ ,$$

then, we have:

$$\Gamma_F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Gamma_{F^{-1}} \ .$$

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**Definition [Carlet, Charpin, Zinoviev, DCC98]**

$F : \mathbb{F}_q \to \mathbb{F}_q$ and $G : \mathbb{F}_q \to \mathbb{F}_q$ are **CCZ-equivalent** if

$$\Gamma_F = \mathcal{L}(\Gamma_G) + c \ .$$

# Advantages of CCZ-equivalence

If $F : \mathbb{F}_q \rightarrow \mathbb{F}_q$ and $G : \mathbb{F}_q \rightarrow \mathbb{F}_q$ are **CCZ-equivalent**. Then

⋆ Differential properties are the same: $\delta_F = \delta_G$ .

Differential uniformity: maximum value of the DDT

$$\delta_F = \max_{a \neq 0, b} |\{x \in \mathbb{F}_q^m, F(x + a) - F(x) = b\}|$$

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⋆ Linear properties are the same: $\mathcal{W}_F = \mathcal{W}_G$ .

Linearity: maximum value of the LAT

$$\mathcal{W}_F = \max_{a, b \neq 0} \left| \sum_{x \in \mathbb{F}_{2^n}^m} (-1)^{a \cdot x + b \cdot F(x)} \right|$$

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- ⋆ The degree is **not preserved**.

Example: in $\mathbb{F}_p$ where

$$p = \texttt{0x73eda753299d7d483339d80809a1d80553bda402fffe5bfeffffffff00000001}$$

if $F(x) = x^5$ then $F^{-1}(x) = x^{5^{-1}}$ where

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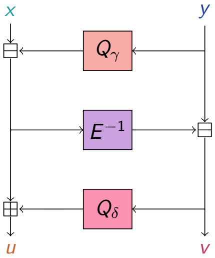
# The Flystel

Butterfly + Feistel ⇒ Flystel

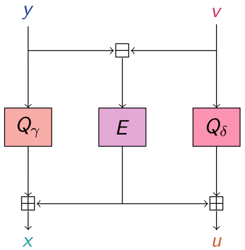
A 3-round Feistel-network with
$Q_\gamma : \mathbb{F}_q \to \mathbb{F}_q$ and $Q_\delta : \mathbb{F}_q \to \mathbb{F}_q$ two quadratic functions, and $E : \mathbb{F}_q \to \mathbb{F}_q$ a permutation

**High-degree** permutation




*Open Flystel* $\mathcal{H}$.

**Low-degree** function




*Closed Flystel* $\mathcal{V}$.

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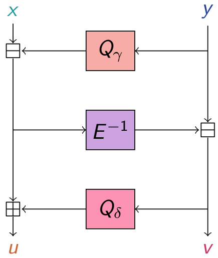


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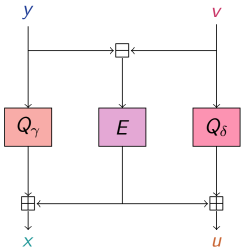


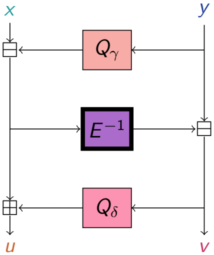

*Closed Flystel $\mathcal{V}$.*

$$\Gamma_{\mathcal{H}} = \mathcal{L}(\Gamma_{\mathcal{V}}) \quad \text{s.t.} \quad ((x, y), (u, v)) = \mathcal{L}\left( ((v, y), (x, u)) \right)$$

# Advantage of CCZ-equivalence

⋆ High Degree Evaluation.

**High-degree**
permutation




*Open* `Flystel` $\mathcal{H}$.

Ex: if $E : x \mapsto x^5$ in $\mathbb{F}_p$ where

$$p = \texttt{0x73eda753299d7d483339d80809a1d805} \\ \texttt{53bda402fffe5bfeffffffff00000001}$$

then $E^{-1} : x \mapsto x^{5^{-1}}$ where

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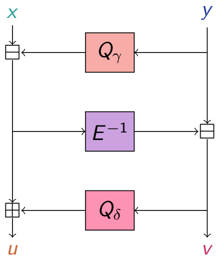
# Advantage of CCZ-equivalence

- ⋆ High Degree Evaluation.
- ⋆ Low Cost Verification.

$$(u, v) == \mathcal{H}(x, y) \Leftrightarrow (x, u) == \mathcal{V}(y, v)$$

**High-degree** permutation

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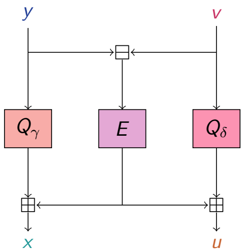
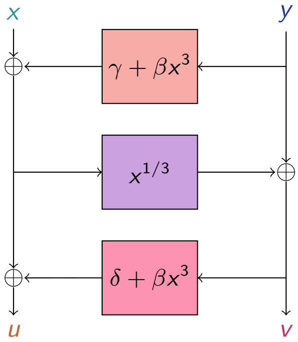


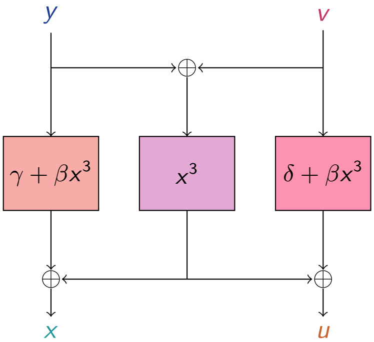

*Closed* `Flystel` $\mathcal{V}$.

# Flystel in $\mathbb{F}_{2^n}$

$$Q_\gamma(x) = \gamma + \beta x^3 , \quad Q_\delta(x) = \delta + \beta x^3 , \quad \text{and} \quad E(x) = x^3$$

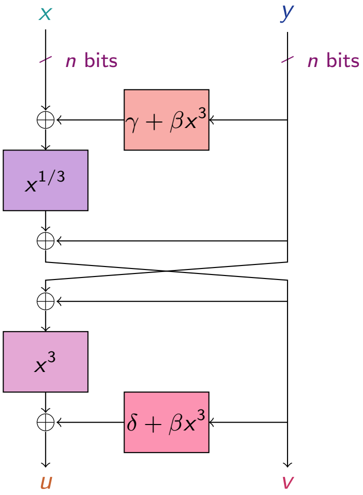



*Open* `Flystel`$_2$.




*Closed* `Flystel`$_2$.

# Properties of Flystel in $\mathbb{F}_{2^n}$




*Degenerated Butterfly.*

Introduced by [Perrin et al. 2016].

Theorems in [Li et al. 2018] state that if $\beta \neq 0$:

- ⋆ Differential properties

$$\delta_{\mathcal{H}} = \delta_{\mathcal{V}} = 4$$

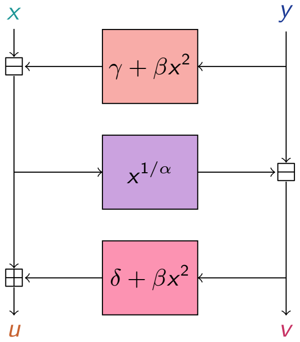
- ⋆ Linear properties

$$\mathcal{W}_{\mathcal{H}} = \mathcal{W}_{\mathcal{V}} = 2^{n+1}$$

- ⋆ Algebraic degree
  - ⋆ Open $\texttt{Flystel}_2$: $\deg_{\mathcal{H}} = n$
  - ⋆ Closed $\texttt{Flystel}_2$: $\deg_{\mathcal{V}} = 2$

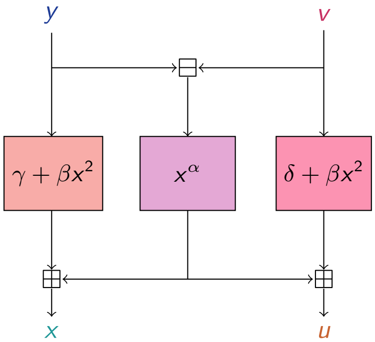
# Flystel in $\mathbb{F}_p$

$$Q_\gamma(x) = \gamma + \beta x^2 \ , \quad Q_\delta(x) = \delta + \beta x^2 \ , \quad \text{and} \quad E(x) = x^\alpha$$




*Open* `Flystel`$_p$.

usually
$\alpha = 3$ or 5.




*Closed* `Flystel`$_p$.

# Properties of Flystel in $\mathbb{F}_p$

⋆ Differential properties

$\texttt{Flystel}_\texttt{p}$ has a differential uniformity:

$$\delta_{\mathcal{H}} = \max_{a \neq 0, b} \left| \{ x \in \mathbb{F}_p^2, \mathcal{H}(x + a) - \mathcal{H}(x) = b \} \right| \leq \alpha - 1$$

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⋆ Linear properties

Conjecture:

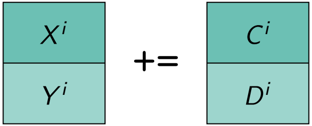
$$\mathcal{W}_{\mathcal{H}} = \max_{a, b \neq 0} \left| \sum_{x \in \mathbb{F}_p^2} exp\left(\frac{2\pi i(\langle a, x\rangle - \langle b, \mathcal{H}(x)\rangle)}{p}\right)\right| \leq p \log p \text{ ?}$$

# The SPN Structure

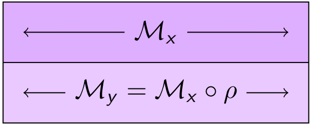
The internal state of `Anemoi` and its basic operations.

A Substitution-Permutation Network with:

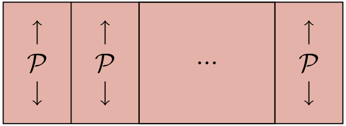

(a) *Internal state.*



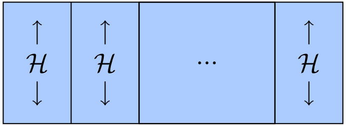

(b) *The constant addition.*




(c) *The diffusion layer.*

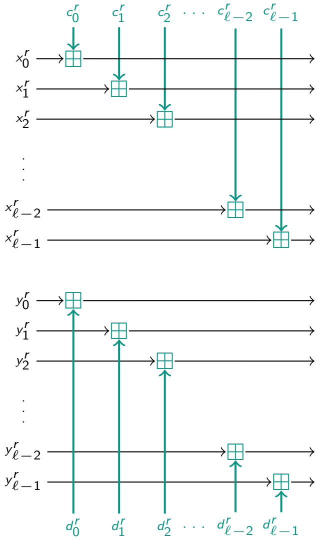



(d) *The Pseudo-Hadamard Transform.*




(e) *The S-box layer.*

# The SPN Structure



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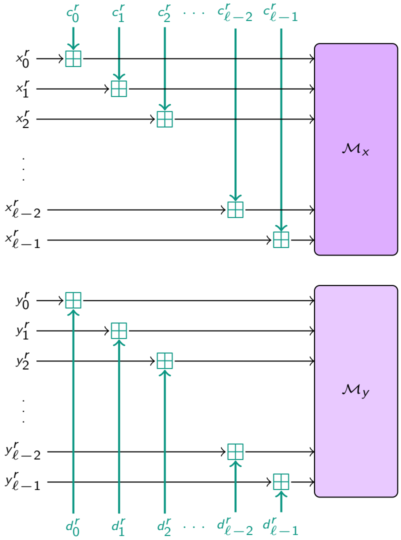

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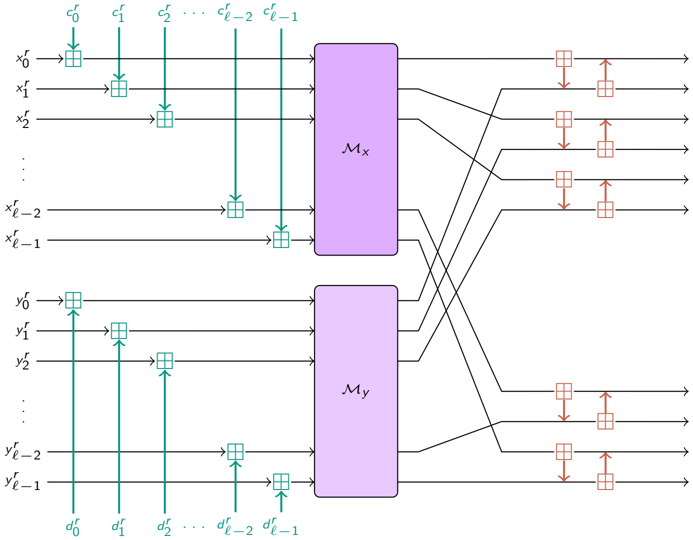

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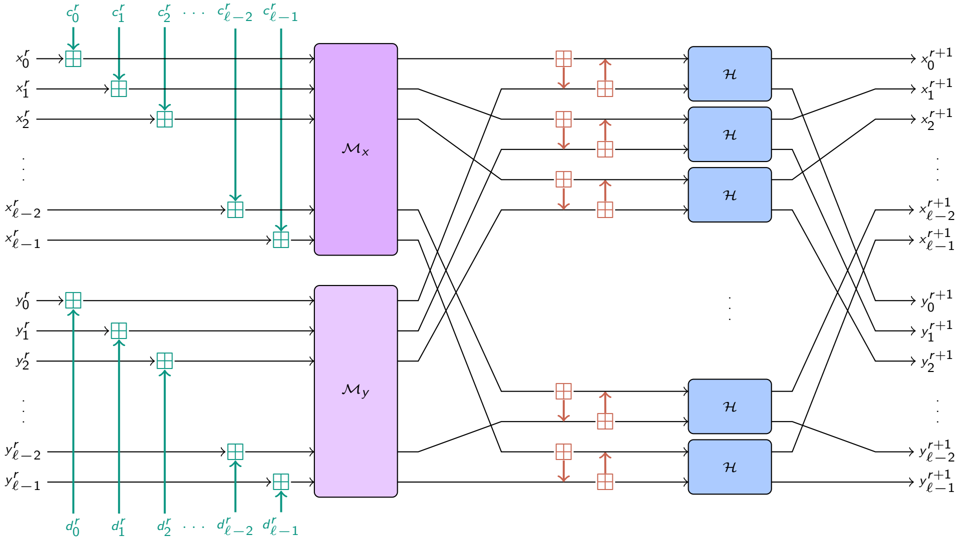

# Number of rounds

$$\texttt{Anemoi}_{q,\alpha,\ell} = \mathcal{M} \circ R_{n_r-1} \circ \ldots \circ R_0$$

⋆ Choosing the number of rounds

$$n_r \geq \max \left\{ 8 \ , \ \underbrace{\min(5, 1+\ell)}_{\text{security margin}} + \underbrace{2 + \min\left\{ r \in \mathbb{N} \ \middle| \ \binom{4\ell r + \kappa_\alpha}{2\ell r}^2 \geq 2^s \right\}}_{\text{to prevent algebraic attacks}} \right\} .$$

| $\alpha$ ($\kappa_\alpha$) | 3 (1) | 5 (2) | 7 (4) | 11 (9) |
|---|---|---|---|---|
| $\ell = 1$ | 21 | 21 | 20 | 19 |
| $\ell = 2$ | 14 | 14 | 13 | 13 |
| $\ell = 3$ | 12 | 12 | 12 | 11 |
| $\ell = 4$ | 12 | 12 | 11 | 11 |

*Number of rounds of Anemoi ($s = 128$).*

# Some Benchmarks

| | $m$ $(= 2\ell)$ | RP[1] | Poseidon[2] | Griffin[3] | Anemoi |
|---|---|---|---|---|---|
| R1CS | 2 | 208 | 198 | - | **76** |
| | 4 | 224 | 232 | 112 | **96** |
| | 6 | 216 | 264 | - | **120** |
| | 8 | 256 | 296 | 176 | **160** |
| Plonk | 2 | 312 | 380 | - | **191** |
| | 4 | 560 | 832 | **260** | 316 |
| | 6 | 756 | 1344 | - | **460** |
| | 8 | 1152 | 1920 | **574** | 648 |
| AIR | 2 | 156 | 300 | - | **126** |
| | 4 | **168** | 348 | **168** | **168** |
| | 6 | **162** | 396 | - | 216 |
| | 8 | **192** | 456 | 264 | 288 |

(a) *when* $\alpha = 3$

| | $m$ $(= 2\ell)$ | RP | Poseidon | Griffin | Anemoi |
|---|---|---|---|---|---|
| R1CS | 2 | 240 | 216 | - | **95** |
| | 4 | 264 | 264 | **110** | 120 |
| | 6 | 288 | 315 | - | **150** |
| | 8 | 384 | 363 | **162** | 200 |
| Plonk | 2 | 320 | 344 | - | **212** |
| | 4 | 528 | 696 | **222** | 344 |
| | 6 | 768 | 1125 | - | **496** |
| | 8 | 1280 | 1609 | **492** | 696 |
| AIR | 2 | **200** | 360 | - | 210 |
| | 4 | **220** | 440 | **220** | 280 |
| | 6 | **240** | 540 | - | 360 |
| | 8 | **320** | 640 | 360 | 480 |

(b) *when* $\alpha = 5$

*Constraint comparison for standard arithmetization, without optimization* $(s = 128)$.

[1] *Rescue* [Aly et al., ToSC 2020]
[2] Poseidon [Grassi et al., USENIX 2021]
[3] Griffin [Grassi et al., CRYPTO 2023]

# Conclusions

Anemoi: A new family of ZK-friendly hash functions

- ⋆ Contributions of fundamental interest:
  - ⋆ New S-box: Flystel
  - ⋆ New mode: Jive
- ⋆ Identify a link between AO and CCZ-equivalence

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Related works

- $\text{AnemoiJive}_3$ with TurboPlonK [Liu et al., 2022]
- Arion [Roy, Steiner and Trevisani, 2023]
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☞ More details on eprint.iacr.org/2022/840 or on anemoi-hash.github.io

# Announcement

## Cryptanalysis and design of symmetric primitives defined over large finite fields

**November 27th, at 2:00pm**

Inria Paris

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Inria Paris

*Thanks for your attention!* 

# More benchmarks and

# Cryptanalysis

# Purposes of Anemoi

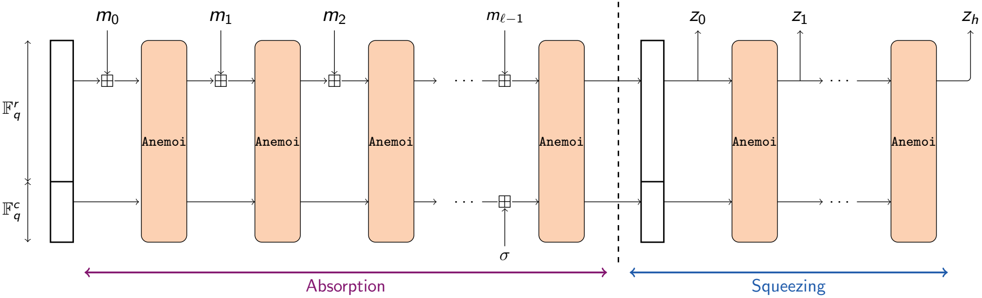
The 2 purposes of Anemoi:

- ⋆ a hash function to emulate a random oracle
- ⋆ a compression function within a Merkle-tree

Using different functions for the different purposes

# Sponge construction

- Hash function (random oracle):
  - input: arbitrary length
  - ouput: fixed length

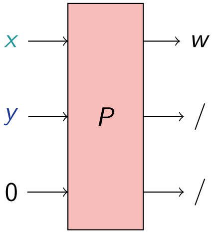


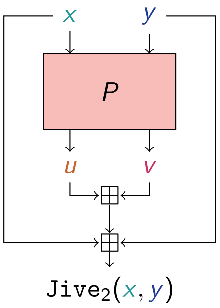
# New Mode: Jive

- Compression function (Merkle-tree):
  - input: fixed length
  - output: (input length) /2

Dedicated mode: 2 words in 1

$$(x, y) \mapsto x + y + u + v \ .$$



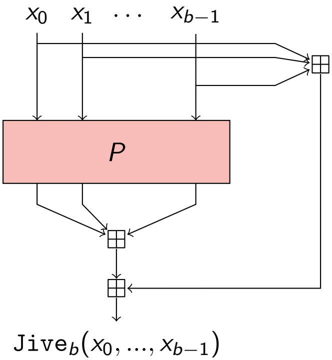



# New Mode: Jive

- ⋆ Compression function (Merkle-tree):
  - ⋆ input: fixed length
  - ⋆ output: (input length) /b

Dedicated mode: b words in 1

$$\texttt{Jive}_b(P) : \begin{cases} (\mathbb{F}_q^m)^b & \to \mathbb{F}_q^m \\ (x_0, \ldots, x_{b-1}) & \mapsto \sum_{i=0}^{b-1} (x_i + P_i(x_0, \ldots, x_{b-1})) \end{cases} .$$



# Comparison for Plonk (with optimizations)

| | $m$ | Constraints |
|---|---|---|
| Poseidon | 3 | 110 |
| | 2 | 88 |
| Reinforced Concrete | 3 | 378 |
| | 2 | 236 |
| Rescue–Prime | 3 | 252 |
| Griffin | 3 | 125 |
| AnemoiJive | 2 | **86** |

(a) *With 3 wires.*

| | $m$ | Constraints |
|---|---|---|
| Poseidon | 3 | 98 |
| | 2 | 82 |
| Reinforced Concrete | 3 | 267 |
| | 2 | 174 |
| Rescue–Prime | 3 | 168 |
| Griffin | 3 | 111 |
| AnemoiJive | 2 | **64** |

(b) *With 4 wires.*

*Constraints comparison with an additional custom gate for $x^\alpha$. ($s = 128$).*

# Comparison for Plonk (with optimizations)

| | $m$ | Constraints |
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| Poseidon | 3<br>2 | 110<br>88 |
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| Rescue–Prime | 3 | 252 |
| Griffin | 3 | 125 |
| AnemoiJive | 2 | ~~**86**~~ **56** |

(a) *With 3 wires.*

| | $m$ | Constraints |
|---|---|---|
| Poseidon | 3<br>2 | 98<br>82 |
| Reinforced Concrete | 3<br>2 | 267<br>174 |
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| Griffin | 3 | 111 |
| AnemoiJive | 2 | **64** |

(b) *With 4 wires.*

*Constraints comparison with an additional custom gate for $x^\alpha$. ($s = 128$).*

**with an additional quadratic custom gate: 56 constraints**

# Native performance

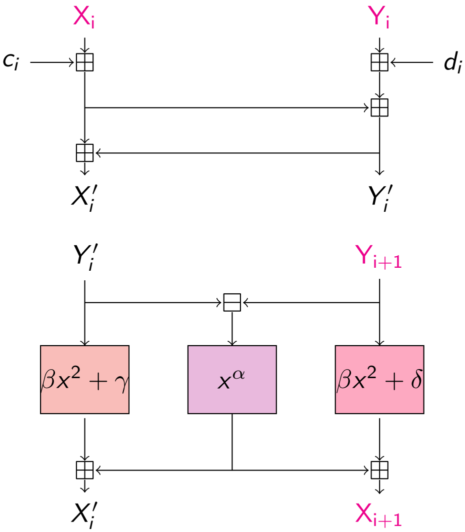
| *Rescue*-12 | *Rescue*-8 | Poseidon-12 | Poseidon-8 | Griffin-12 | Griffin-8 | Anemoi-8 |
|---|---|---|---|---|---|---|
| 15.67 $\mu$s | 9.13 $\mu$s | 5.87 $\mu$s | 2.69 $\mu$s | 2.87 $\mu$s | **2.59 $\mu$s** | 4.21 $\mu$s |

*2-to-1 compression functions for $\mathbb{F}_p$ with $p = 2^{64} - 2^{32} + 1$ ($s = 128$).*

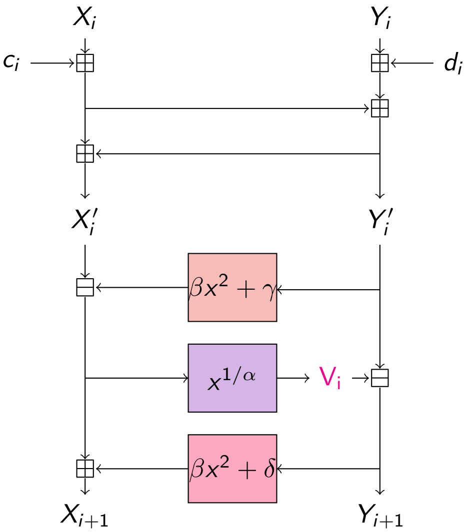
| *Rescue* | Poseidon | Griffin | Anemoi |
|---|---|---|---|
| 206 $\mu$s | **9.2 $\mu$s** | 74.18 $\mu$s | 128.29 $\mu$s |

*For $BLS12-381$, Rescue, Poseidon, Anemoi with state size of 2, Griffin of 3 ($s = 128$).*

# Algebraic attacks: 2 modelings




(a) *Model 1.*

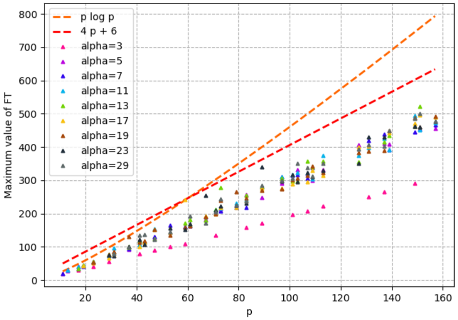



(b) *Model 2.*

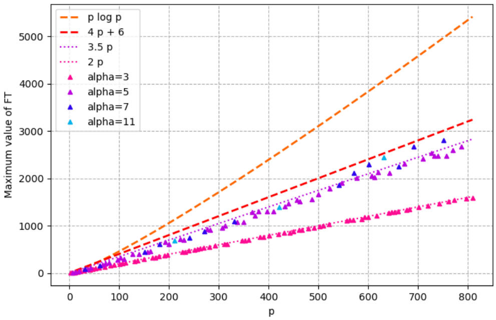
# Properties of Flystel in $\mathbb{F}_p$

⋆ Linear properties

$$\mathcal{W}_{\mathcal{H}} = \max_{a,b \neq 0} \left| \sum_{x \in \mathbb{F}_p^2} exp\left( \frac{2\pi i(\langle a, x\rangle - \langle b, \mathcal{H}(x)\rangle)}{p} \right) \right| \leq p \log p \ ?$$




(a) *For different $\alpha$.*



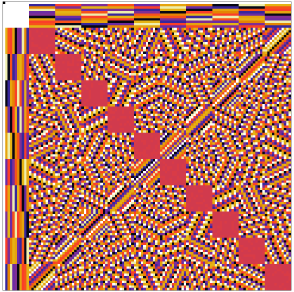

(b) *For the smallest $\alpha$.*

*Conjecture for the linearity.*

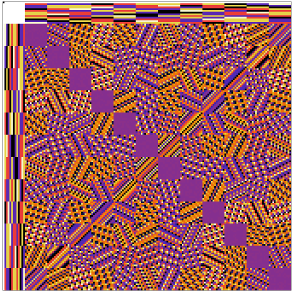
# Properties of Flystel in $\mathbb{F}_p$

- Linear properties

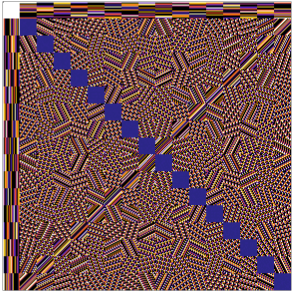
$$\mathcal{W}_{\mathcal{H}} = \max_{a,b \neq 0} \left| \sum_{x \in \mathbb{F}_p^2} exp\left( \frac{2\pi i(\langle a, x\rangle - \langle b, \mathcal{H}(x)\rangle)}{p} \right) \right| \leq p \log p \text{ ?}$$



(a) *when* $p = 11$ *and* $\alpha = 3$.



(b) *when* $p = 13$ *and* $\alpha = 5$.



(c) *when* $p = 17$ *and* $\alpha = 3$.

*LAT of* $\texttt{Flystel}_p$.